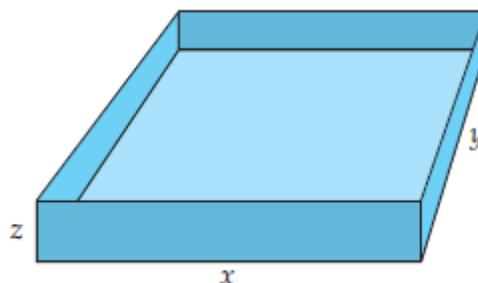


## Modeling with Polynomials

### MAT 142, Spring 2017

A classic problem in modeling with polynomials is the open-top box. The problem involves determine what size box can be made from a flat sheet of a given size, by cutting out corners from the sheet and folding up the sides. Students are often then asked to determine properties of the box they create, like the maximum volume or the total cost of the surface, etc.



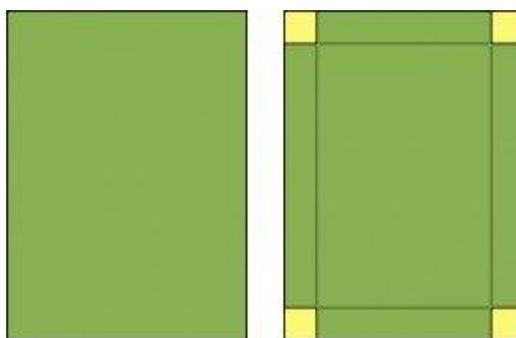
This project will construct a function for the volume using modeling techniques and compare the resulting function to the one we obtain from using algebraic principles.

It is intended for this project to be done in small groups, but individual students can successfully complete the tasks below. During class time, however, I encourage students not to work alone. A class can elect to collect data and share it with the class to make the process go more quickly.

Materials needed:

- A piece of paper cut to a square
- A piece of paper cut to a rectangle
- A pair of scissors
- A ruler or other measuring device
- A calculator (a graphing calculator with regression features, or use a spreadsheet program like Excel)
- Colored pencils

A. Beginning with the square sheet, cut small squares from the corner as shown below. (Start with very small squares. It's easy to make the squares you cut out bigger; it's not so easy to put material back!)



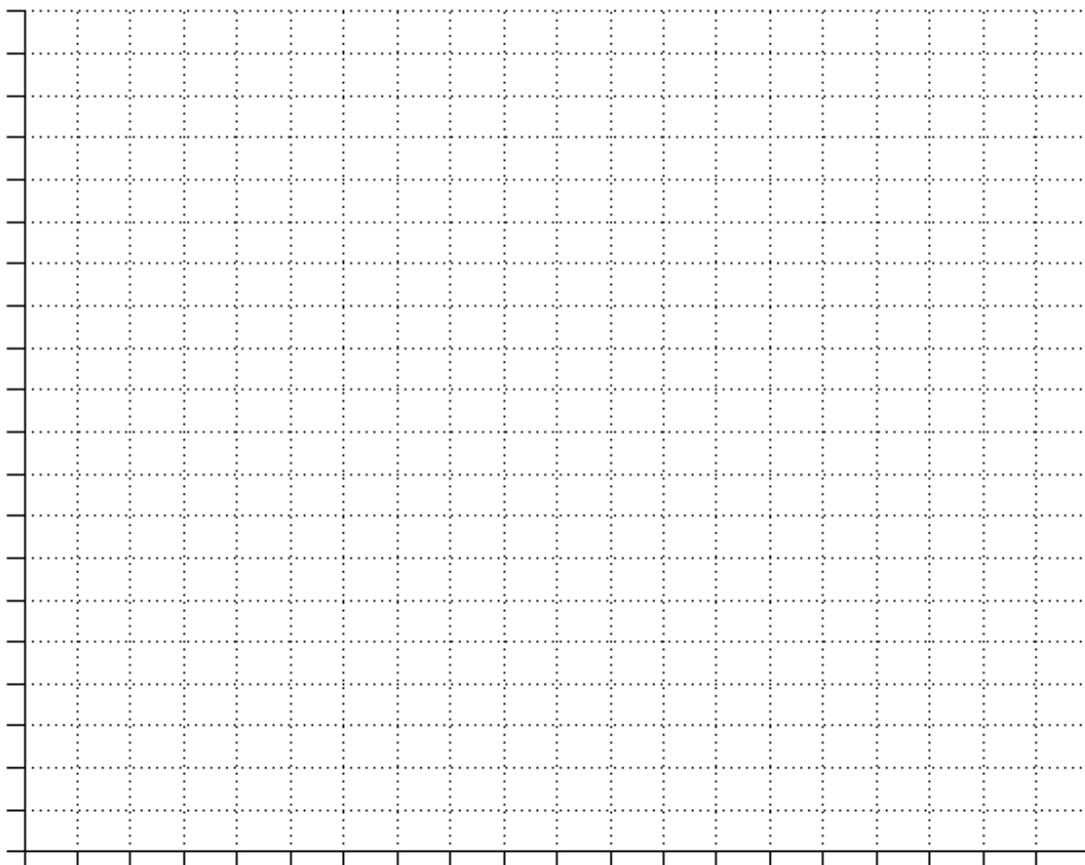
B. Fold up the edges to create a box that looks like the one at the top of the page.

C. Use your ruler to measure the dimensions of the resulting box and record your measurements in the table below. Calculate the volume and record it in the last column.

It may help to label on your sheet which directions is width and which is length so that on your next measurement, you can be consistent with which is which.

D. Repeat your calculation until the cuts you are making use up nearly all the material and the volume of your box is shrinking.



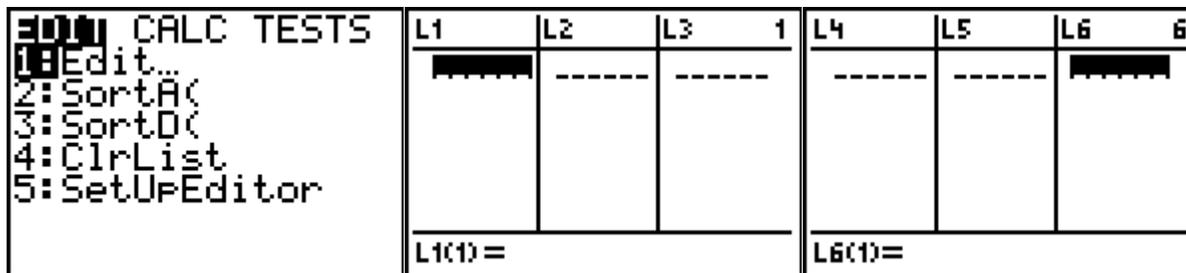


*Check with your instructor.*

- G. Our next step is to construct a regression equation in our calculator to model the volume of the box. Follow the steps below to enter data into a TI-84.



Press the **STAT** key and then select **Edit**. You will now see the data screen with the six lists, L<sub>1</sub> through L<sub>6</sub> (use your right arrow to see L<sub>4</sub> through L<sub>6</sub>).



If there is already data in one of the lists, we need to clear the list. Do this by placing your cursor at the top of the list you wish to clear and then press the **CLEAR** button. DO NOT press the **DEL** button - this will delete the list from your calculator.

Enter your  $x$  data into  $L_1$  and your volume data into  $L_2$ . You can navigate from list to list with the cursor arrow buttons.

H. Sketch the graph in your calculator. To draw the graph in the TI, select **2nd** **Y=** to enter **StatPlot**. Turn on Plot1, press **ENTER**, and move the cursor to ON and press **ENTER** again. On the Type line, the first graph type is the scatterplot. Put the cursor over it and press **ENTER** to select. We used the default XList ( $L_1$ ) and the default YList ( $L_2$ ), so we don't need to adjust those settings.

After setting up the graphing screen, press **ZOOM**, followed by **9** for **9: ZoomStat**. This will adjust your Window settings to match the data sorted in your lists.

```

Plot1 Plot2 Plot3
On Off
Type: [Scatter] [Line] [Bar] [Histogram] [Normal] [Log]
Xlist: L1
Ylist: L2
Mark: [Square] + . .
Color: BLUE
  
```

I. Compare the results to the scatterplot you drew above. Does it match? (If the answer is 'no', find the error before proceeding!)

J. Next we need to set up the calculator to find the regression equation for us. Complete the following steps.

1. Turn on Diagnostics:

a. On newer TI-84s, select **MODE** and scroll down toward the bottom on the list of commands (onto the second page). You will see a line for the Diagnostics. Select ON to be highlighted.

b. On older machines (TI-83s, and some older 84s), press **2nd** **0** for **CATALOG**. Scroll down through the alphabetical list to the line that reads Diagnostics

On. Select **ENTER** when you get to that line and then press **ENTER** again to execute the command.  
 (You should only have to do this once unless someone clears the memory.)

2. Select **STAT** and then scroll over the **CALC** menu. Scroll down to **6: CubicReg**. This is the linear regression function. Select **ENTER** or **6**. On newer TI-84s, your screen will look like this:

```

CubicReg
Xlist: 1
Ylist: L2
FreqList:
Store RegEQ:
Calculate
  
```

Accept the defaults for the List locations unless you used a different list. Leave the frequency line blank. On the Store RegEQ

line type: **VAR** then select **Y-VARS**, then **FUNCTION**, then **Y1**. Then scroll

down to Calculate and press **ENTER**.

On older machines your screen will look like this:

```

CubicReg L1,L2,Y1
  
```

Before hitting anything else, follow the steps above to select  $Y_1$  regression equation

location. Then press **ENTER** to execute the command.

Your screen will look something like what is shown here, though, with different numbers.

The equation the coefficients belong to appears first:  $y = ax^3 + bx^2 + cx + d$ , and the values of the coefficients are shown below in order. The final value show up only when the Diagnostic setting is ON. The  $R^2$  measures how good the fit to the data is. Closer to 1 is a better fit.

```

CubicReg
y=ax3+bx2+cx+d
a=-.0498363582
b=2.350568897
c=-7.510077028
d=26.64954657
R2=.9876878531
  
```

- K. What equation did your calculator give you? Write the result in function notation.

**Check with your instructor.**

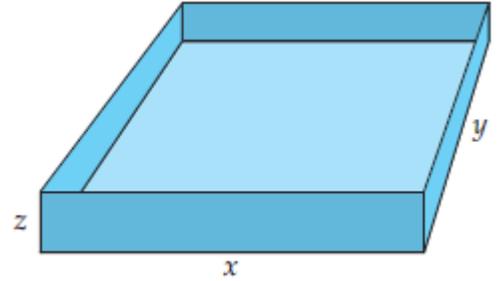
- L. Graph the equation against your plotted data. Visually inspect how good the fit is. Does the equation do a good job of modeling the data you have? If it does not, consult with your instructor about obtaining a different equation type. Add your final equation to the scatterplot in F.

- M. Next we want to compare our equation to what we might obtain from algebraic principles. Return to the expressions you came up with in E for Length, Width and Height, and multiply them to obtain a volume equation. Multiply out the factored expression.
- N. How do the coefficients of the equation above (M) match the coefficients of the equation you obtained from the calculator in K?
- O. Add your algebraic function to your calculator in  $Y_2$  and compare the equation to the data. Add it to the scatterplot in F with a different color pencil.
- P. What is the domain and range of your volume equation? Use your graph and explain your reasoning. [Hint: What happens to the volume if you go outside the domain? Is this realistic?]
- Q. How do your considerations of domain and range change knowing that this is a real-world problem and not simply a random algebraic function?
- R. Repeat the above process with the rectangular sheet of paper. Record values in the table below.





- W. If we wanted instead to obtain the surface area equation (rather than the volume equation), what would need to change? Write an expression for the surface area of your open-top box based on the size of square removed from the corner.



- X. If the cost of the material for the box is \$6.50 per square inch, how much material is needed to construct the box of largest volume (using your equation from V) if we neglect the material removed from the corners?

***Check with your instructor.***