MAT 142 Homework #8 Key

1a. $\tan x \csc x \cos x = 1$ (prove)

$\frac{\sin x}{\cos x} \cdot \frac{1}{\sin x} \cdot \cos x = 1$

b. $\frac{\cos\theta \sec\theta}{\cot\theta} = \frac{\cos\theta \cdot \frac{1}{\cos\theta}}{\cot\theta} = \frac{1}{\cot\theta} = \tan\theta$

c. $\frac{\sin t}{\tan t} + \frac{\cos t}{\cot t} = \frac{\sin t}{\left(\frac{\sin t}{\cos t}\right)} + \frac{\cos t}{\left(\frac{\cos t}{\sin t}\right)} = \sin t\left(\frac{\cos t}{\sin t}\right) + \cos t\left(\frac{\sin t}{\cos t}\right)$

$= \cos t + \sin t$

d. $\frac{\sec x - \csc x}{\sec x + \csc x} \cdot \frac{\sin x + \cos x}{\sin x + \cos x} = \frac{\frac{\sin x}{\cos x} + 1 - 1 - \frac{\cos x}{\sin x}}{\frac{\sin x}{\cos x} + 1 + 1 + \frac{\cos x}{\sin x}} =$

$\frac{\tan x - \cot x}{\tan x + 2 + \cot x} \cdot \frac{\tan x}{\tan x} = \frac{\tan^2 x - 1}{\tan^2 x + 2\tan x + 1} = \frac{(\tan x + 1)(\tan x - 1)}{(\tan x + 1)(\tan x + 1)}$

$= \frac{\tan x - 1}{\tan x + 1}$

e. $\frac{1 + \cos t}{1 - \cos t} \frac{\frac{1}{\sin t}}{\frac{1}{\sin t}} = \frac{\csc t + \cot t}{\csc t - \cot x} \cdot \frac{\csc t + \cot t}{\csc t + \cot t} = \frac{(\csc t + \cot t)^2}{\csc^2 t - \cot^2 t}$ ($\csc^2 t - \cot^2 t = 1$)

$= (\csc t + \cot t)^2$

f. $(\tan^2\theta + 1)(\cos^2 t + 1) = \frac{\sin^2 t}{\cos^2 t}\cos^2 t + \tan^2 t + \cos^2 t + 1 =$

$\sin^2 t + \cos^2 t + \tan^2 t + 1 = 1 + \tan^2 t + 1 = \tan^2 t + 2$