Multiple Variable Differentiation Review Key

1) $f(x,y) = e^x \cos(y)$

$f_x = e^x \cos(y) \qquad f_y = -e^x \sin(y)$

2) $g(x,y) = \frac{xy}{x+y}$

$g_x = \frac{y(x+y) - 1(xy)}{(x+y)^2} = \frac{\cancel{xy} + y^2 - \cancel{xy}}{(x+y)^2} = \frac{y^2}{(x+y)^2}$

$g_y = \frac{x(x+y) - 1(xy)}{(x+y)^2} = \frac{x^2 + \cancel{xy} - \cancel{xy}}{(x+y)^2} = \frac{x^2}{(x+y)^2}$

3) $z(x,y) = \ln(x^2+y^2+1)$

$z_x = \frac{1}{x^2+y^2+1} \cdot 2x = \frac{2x}{x^2+y^2+1} \qquad z_y = \frac{1}{x^2+y^2+1} \cdot 2y = \frac{2y}{x^2+y^2+1}$

4) $m(x,y,z) = z \arctan\left(\frac{y}{x}\right)$

$m_x = z \cdot \frac{1}{1+\left(\frac{y}{x}\right)^2} \cdot \frac{-y}{x^2} = \frac{-zy}{x^2+y^2} \qquad m_y = z \cdot \frac{1}{1+\left(\frac{y}{x}\right)^2} \cdot \frac{1}{x} \cdot \frac{x}{x} = \frac{zx}{x^2+y^2}$

$m_z = \arctan(y/x)$

5) $u(x,t) = ce^{-n^2t} \sin(nx)$ note: c, n are constants

$u_x = ce^{-n^2t} \cos(nx) \cdot n = cne^{-n^2t} \cos(nx)$

$u_t = ce^{-n^2t} \cdot (-n^2) \sin(nx) = -cn^2e^{-n^2t} \sin(nx)$

6) $v(x,t) = c \sinh(kx) \cos(kt)$

$v_x = ck \cosh(kx) \cos(kt)$

$v_t = -ck \sinh(kx) \sin(kt)$

7) $g(r,s,t) = r^2e^{-st} \sin(rs)$

$g_r = 2re^{-st} \sin(rs) + r^2e^{-st} \cos(rs) \cdot s = re^{-st}(2\sin(rs) + rs\cos(rs))$

$g_s = r^2e^{-st}(-t) \sin(rs) + r^2e^{-st} \cos(rs) \cdot r = r^2e^{-st}(r\cos(rs) - t\sin(rs))$

$g_t = r^2 e^{-st}(-s) \sin(rs) = -r^2se^{-st} \sin(rs)$

8) $r(u,v) = u^3 - 3uv + v^2$

$r_u = 3u^2 - 3v \qquad r_v = -3u + 2v$

9) $p(x,y) = \arctan\left(\frac{y}{x}\right) - \ln\sqrt{x^2+y^2} = \arctan\left(\frac{y}{x}\right) - \frac{1}{2}\ln(x^2+y^2)$

$$P_x = \frac{1}{1+\left(\frac{y}{x}\right)^2}\cdot\frac{-y}{x^2} - \frac{1}{2}\frac{1}{x^2+y^2}\cdot 2x = \frac{-y}{x^2+y^2} - \frac{x}{x^2+y^2} = \frac{-y-x}{x^2+y^2}$$

$$P_y = \frac{1}{1+(y/x)^2}\cdot\frac{1}{x}\cdot\frac{x}{x} - \frac{1}{2}\frac{1}{x^2+y^2}\cdot 2y = \frac{x}{x^2+y^2} - \frac{y}{x^2+y^2} = \frac{x-y}{x^2+y^2}$$

10) $w(x,y) = \arccos(xy)$

$$w_x = \frac{1}{\sqrt{1-x^2y^2}}\cdot y = \frac{y}{\sqrt{1-x^2y^2}} \qquad w_y = \frac{1}{\sqrt{1-x^2y^2}}\cdot x = \frac{x}{\sqrt{1-x^2y^2}}$$

11) $F(x,y) = \int_x^y t^2 - 1\,dt = -\int_y^x t^2 - 1\,dt$

using second fundamental theorem of calculus:

$F_x = -(x^2-1) = 1 - x^2$

$F_y = y^2 - 1$