

MTH 291 Skills #8 key

(1)

1a. $6y'' - 5y' + y = 0$

$6r^2 - 5r + 1 = 0$

$(3r-1)(2r-1) = 0$

$r = 1/3, r = 1/2$

$y = c_1 e^{1/3t} + c_2 e^{1/2t}$

$\rightarrow 4 = c_1 + c_2$

$y' = 1/3 c_1 e^{1/3t} + 1/2 c_2 e^{1/2t} \rightarrow 0 = 1/3 c_1 + 1/2 c_2$

$c_1 = 12, c_2 = -8$

$y = 12e^{1/3t} - 8e^{1/2t}$

as $t \rightarrow \infty, y \rightarrow -\infty$

max at $t = 0$.

$(0, 4)$

b. $2y'' - 3y' + y = 0$

$2r^2 - 3r + 1 = 0$

$(2r-1)(r-1) = 0$

$r = 1/2, r = 1$

$y = c_1 e^{1/2t} + c_2 e^t$

$\rightarrow 2 = c_1 + c_2$

$y' = 1/2 c_1 e^{1/2t} + c_2 e^t$

$\rightarrow 1/2 = 1/2 c_1 + c_2$

$c_1 = 3, c_2 = -1$

$y = 3e^{1/2t} - e^t$

as $t \rightarrow \infty, y \rightarrow -\infty$

max at $\approx t = .810931, y = 2.25$

c. $y'' + 4y' + 5y = 0$

$r^2 + 4r + 5 = 0$

$r = \frac{-4 \pm \sqrt{16 - 20}}{2} = -2 \pm i$

$e^{(-2+i)t} = e^{-2t} (\cos t + i \sin t) \rightarrow$

$y = c_1 e^{-2t} \cos t + c_2 e^{-2t} \sin t$

$1 = c_1$

$y' = -2e^{-2t} \cos t - e^{-2t} \sin t - 2c_2 e^{-2t} \sin t + c_2 e^{-2t} \cos t = 0$

$-2e^{-2t} \cos t + c_2 e^{-2t} \cos t = 0 \Rightarrow c_2 = 2$

$y = e^{-2t} \cos t + 2e^{-2t} \sin t$

as $t \rightarrow \infty, y \rightarrow 0$

∞ # of critical points