MTH 291 Skills Problems #3 Key

1. it is linear since $\frac{d^3y}{dt^3}, \frac{dy}{dt}, y$ are not raised to powers, multiply each other, etc. 3rd order. ordinary.

2. a. $y' - 2y = 3e^t$ $\quad \mu = e^{\int -2\,dt} = e^{-2t}$

$e^{-2t}y' - 2e^{-2t}y = 3e^t e^{-2t} = 3e^{-t}$

$\int (ye^{-2t})' = \int 3e^{-t}\,dt \rightarrow ye^{-2t} = -3e^{-t} + C$

$y = -3e^{t} + Ce^{2t}$

b. $ty' - y = -t^2e^{-t} \rightarrow y' - \frac{1}{t}y = te^{-t}$

$\frac{1}{t}y' - \frac{1}{t^2}y = e^{-t}$ $\quad \mu = e^{\int -\frac{1}{t}dt} = e^{-\ln t} = e^{\ln \frac{1}{t}} = \frac{1}{t}$

$\int \left(y \cdot \frac{1}{t}\right)' = \int e^{-t}\,dt \rightarrow y \cdot \frac{1}{t} = -e^{-t} + C$

$y = -te^{-t} + Ct$

c. $t^3y' + 4t^2y = e^{-t}$ $\quad y(-1) = 0$

$y' + \frac{4}{t}y = \frac{e^{-t}}{t^3}$ $\quad \mu = e^{\int \frac{4}{t}dt} = e^{4\ln t} = e^{\ln t^4} = t^4$

$t^4y' + 4t^3y = te^{-t}$

$\int (t^4y)' = \int te^{-t}\,dt$

$u = t \quad du = dt$

$dv = e^{-t} \quad v = -e^{-t}$

$t^4y = -te^{-t} + \int e^{-t}\,dt$

$= -te^{-t} - e^{-t} + C$

$y = \frac{-e^{-t}}{t^3} - \frac{e^{-t}}{t^4} + \frac{C}{t^4}$

$0 = \frac{+e}{-1} - \frac{e}{1} + \frac{C}{1} \Rightarrow C = 0$

$y = \frac{-e^{-t}}{t^3} - \frac{e^{-t}}{t^4}$