

KEY

Instructions: Show all work. Use exact answers unless specifically asked to round.

1. Consider the solutions to a second order differential equation in the table below, and the given forcing function. What is the appropriate Ansatz for the method of undetermined coefficients?

	$y_1(x)$	$y_2(x)$	$f(x)$	$Y(x)$
a.	e^x	e^{6x}	$\frac{1}{2}\sin(4x)$	$A\sin 4x + B\cos 4x$
b.	$\sin(2x)$	$\cos(2x)$	$2xe^x + \cos x$	$Axe^x + Be^x + C\cos x + D\sin x$
c.	$e^{-x}\cos 3x$	$e^{-x}\sin 3x$	$9\sin 3x$	$A\sin 3x + B\cos 3x$
d.	e^{-2x}	e^{-x}	$0.1e^{-2x}$	Axe^{-2x}

2. Consider the differential equation $y'' + 5y' + 6y = 3\sin 2t - \cos 2t$. Find the particular solution to the non-homogenous differential equation. Find all undetermined coefficients.

$$r^2 + 5r + 6 = 0$$

$$(r+2)(r+3) = 0$$

$$r = -2, -3$$

$$y_g(t) = c_1 e^{-2t} + c_2 e^{-3t}$$

$$Y(t) = A\sin 2t + B\cos 2t$$

$$Y'(t) = 2A\cos 2t - 2B\sin 2t$$

$$Y''(t) = -4A\sin 2t - 4B\cos 2t$$

$$\begin{aligned} -4A\sin 2t - 4B\cos 2t + 5(2A\cos 2t - 2B\sin 2t) + 6(A\sin 2t + B\cos 2t) \\ = 3\sin 2t - \cos 2t \end{aligned}$$

$$\begin{aligned} -4A\sin 2t - 4B\cos 2t + 10A\cos 2t - 10B\sin 2t + 6A\sin 2t + 6B\cos 2t \\ = 3\sin 2t - \cos 2t \end{aligned}$$

$$\sin 2t : 2A - 10B = 3 \quad A = -\frac{1}{26}$$

$$\cos 2t : 10A + 2B = -1 \quad B = -\frac{4}{13}$$

$$y_p(t) = c_1 e^{-2t} + c_2 e^{-3t} - \frac{1}{26}\sin 2t - \frac{4}{13}\cos 2t$$