MTH 291, Quiz #2, Fall 2017 Name KEY

Instructions: Show all work. Use exact answers unless specifically asked to round.

1. Solve the linear equation $x\frac{dy}{dx} + 4y = x^3 - x$ for the general solution using the method of integrating factors.

$\frac{dy}{dx} + \frac{4}{x}y = x^2 - 1$ $\mu = e^{4\ln x} = x^4$

$x^4\frac{dy}{dx} + 4x^3 dy = x^4(x^2-1)$

$\int (x^4 y)' = \int x^6 - x^4\, dx$

$x^4 y = \frac{1}{7}x^7 - \frac{1}{5}x^5 + C$

$y = \frac{1}{7}x^3 - \frac{1}{5}x + \frac{C}{x^4}$

2. Rewrite the Bernoulli equation $\frac{dy}{d\cancel{x}} - y = e^x y^2$ as a linear equation.

$y^n = y^2 \quad n=2$

$(1-n)y^{-n} = (-1)y^{-2}$

$-1y^{-2}\frac{dy}{dx} + y^{-1} = -e^x$

$z = y^{-1}$

$dz = -y^{-2}\frac{dy}{dx}$

$\boxed{\frac{dz}{dx} + z = -e^x}$

3. A tank with 1000 gallons maximum capacity has 400 gallons of water in it at $t = 0$. Brine solution is pouring into the tank at a rate of 4 gallons per minute, with 4 grams per gallon of salt. The well-mixed solution is draining out of the tank at a rate of 2 gallons per minute. Write the differential equation that models this scenario. For how long will this equation apply?

$\frac{dS}{dt} = \frac{4\text{gal}}{\text{min}} \frac{4\text{grams}}{\text{gal}} - \frac{S}{400+2t\ \text{gal}} \cdot \frac{2\text{gal}}{\text{min}}$

$\frac{dS}{dt} = 16 - \frac{S}{200+t}$

applies until tank fills

300 minutes later