

1a. $\begin{bmatrix} 3 & 6 & | & -3 \\ 5 & 7 & | & 10 \end{bmatrix} \xrightarrow{\frac{1}{3}R_1 \rightarrow R_1} \begin{bmatrix} 1 & 2 & | & -1 \\ 5 & 7 & | & 10 \end{bmatrix} \xrightarrow{-5R_1 + R_2 \rightarrow R_2}$

$\begin{bmatrix} 1 & 2 & | & -1 \\ 0 & -3 & | & 15 \end{bmatrix} \xrightarrow{-\frac{1}{3}R_2 \rightarrow R_2} \begin{bmatrix} 1 & 2 & | & -1 \\ 0 & 1 & | & -5 \end{bmatrix} \xrightarrow{-2R_2 + R_1 \rightarrow R_1}$

$\begin{bmatrix} 1 & 0 & | & 9 \\ 0 & 1 & | & -5 \end{bmatrix} \rightarrow X = \begin{bmatrix} 9 \\ -5 \end{bmatrix}$ Consistent independent

b. $\begin{bmatrix} 2 & 0 & 0 & -4 & | & -10 \\ 0 & 3 & 3 & 0 & | & 0 \\ 0 & 0 & 1 & 4 & | & -1 \\ -3 & 2 & 3 & 1 & | & 5 \end{bmatrix} \xrightarrow{rref} \begin{bmatrix} -3 \\ 5 \\ -5 \\ 1 \end{bmatrix}$ Consistent independent

c. $\begin{bmatrix} 1 & -2 & -3 & | & 0 \\ 0 & 1 & 2 & | & 0 \\ 2 & -4 & 9 & | & 0 \end{bmatrix} \xrightarrow{rref} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ Consistent independent

d. $\begin{bmatrix} 2 & 0 & -6 & | & -8 \\ 0 & 1 & 2 & | & 3 \\ 3 & 6 & -2 & | & -4 \end{bmatrix} \xrightarrow{rref} \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix}$ Consistent independent

e. $\begin{bmatrix} 2 & -5 & 8 & | & 0 \\ -3 & -4 & 2 & | & 0 \end{bmatrix} \xrightarrow{rref} \begin{bmatrix} 1 & 0 & 22/23 & | & 0 \\ 0 & 1 & -28/23 & | & 0 \end{bmatrix}$ Consistent dependent

$X_1 + 22/23 X_3 = 0$
 $X_2 - 28/23 X_3 = 0 \rightarrow X_1 = -22/23 X_3$
 $X_2 = 28/23 X_3$
 $X_3 = X_3$
 $X = \begin{bmatrix} -22 \\ 28 \\ 23 \end{bmatrix} t$

f. $\begin{bmatrix} 2 & 0 & 0 & -4 & 1 & | & 0 \\ 0 & 3 & 3 & 0 & -1 & | & 0 \\ 0 & 0 & 1 & 4 & 6 & | & 0 \\ -3 & 2 & 3 & 1 & -2 & | & 0 \end{bmatrix} \xrightarrow{rref} \begin{bmatrix} 1 & 0 & 0 & 0 & 97/54 & | & 0 \\ 0 & 1 & 0 & 0 & -101/27 & | & 0 \\ 0 & 0 & 1 & 0 & 92/27 & | & 0 \\ 0 & 0 & 0 & 1 & 35/54 & | & 0 \end{bmatrix}$

$X_1 + 97/54 X_5 = 0$
 $X_2 - 101/27 X_5 = 0$
 $X_3 + 92/27 X_5 = 0$
 $X_4 + 35/54 X_5 = 0$
 $X_5 = X_5$
 $X_1 = -97/54 X_5$
 $X_2 = 101/27 X_5$
 $X_3 = -92/27 X_5$
 $X_4 = -35/54 X_5$
 $X_5 = X_5$
 $X = \begin{bmatrix} -97 \\ 202 \\ -184 \\ -35 \\ 54 \end{bmatrix} t$

Consistent dependent

$$2a. \begin{cases} 12a - 12b = 7 \\ 3a + 4b = 0 \end{cases} \quad \left[\begin{array}{cc|c} 12 & -12 & 7 \\ 3 & 4 & 0 \end{array} \right] \quad -4R_2 + R_1 \rightarrow R_2$$

$$\left[\begin{array}{cc|c} 12 & -12 & 7 \\ 0 & -28 & 7 \end{array} \right] \cdot \frac{-1}{28} R_2 \rightarrow R_2 \quad \left[\begin{array}{cc|c} 12 & -12 & 7 \\ 0 & 1 & -1/4 \end{array} \right] \quad \frac{1}{12} R_1 \rightarrow R_1$$

$$\left[\begin{array}{cc|c} 1 & -1 & 7/12 \\ 0 & 1 & -1/4 \end{array} \right] \quad R_1 + R_2 \rightarrow R_1 \quad \left[\begin{array}{cc|c} 1 & 0 & 1/3 \\ 0 & 1 & -1/4 \end{array} \right] \quad \begin{matrix} a = 1/3 \rightarrow x = 3 \\ b = -1/4 \rightarrow y = -4 \end{matrix}$$

$$b. \begin{cases} 2a + b - 2c = 5 \\ 3a - 4b = -1 \\ 2a + b + 3c = 0 \end{cases} \quad \left[\begin{array}{ccc|c} 2 & 1 & -2 & 5 \\ 3 & -4 & 0 & -1 \\ 2 & 1 & 3 & 0 \end{array} \right] \rightarrow \text{rref} \rightarrow \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$$

$$\begin{matrix} a = 1 & \rightarrow & x = 1 \\ b = 1 & \rightarrow & y = 1 \\ c = -1 & \rightarrow & z = -1 \end{matrix}$$

$$3a. \begin{cases} a(1)^3 + b(1)^2 + c(1) + d = 7 \\ a(2)^3 + b(2)^2 + c(2) + d = 17 \\ a(3)^3 + b(3)^2 + c(3) + d = 31 \\ a(4)^3 + b(4)^2 + c(4) + d = 65 \end{cases} \quad \begin{matrix} a + b + c + d = 7 \\ 8a + 4b + 2c + d = 17 \\ 27a + 9b + 3c + d = 31 \\ 64a + 16b + 4c + d = 65 \end{matrix}$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 7 \\ 8 & 4 & 2 & 1 & 17 \\ 27 & 9 & 3 & 1 & 31 \\ 64 & 16 & 4 & 1 & 65 \end{array} \right] \rightarrow \text{rref} \quad \begin{bmatrix} 8/3 \\ -14 \\ 100/3 \\ -15 \end{bmatrix}$$

$$8/3 x^3 - 14x^2 + 100/3 x - 15 = y$$

$$b. \begin{cases} a(-2)^4 + b(-2)^3 + c(-2)^2 + d(-2) + e = 28 \\ a(-1)^4 + b(-1)^3 + c(-1)^2 + d(-1) + e = 0 \\ a(0)^4 + b(0)^3 + c(0)^2 + d(0) + e = -6 \\ a(1)^4 + b(1)^3 + c(1)^2 + d(1) + e = -8 \\ a(2)^4 + b(2)^3 + c(2)^2 + d(2) + e = 0 \end{cases}$$

$$\begin{matrix} 16a - 8b + 4c - 2d + e = 28 \\ a - b + c - d + e = 0 \\ e = -6 \end{matrix}$$

$$\begin{matrix} a + b + c + d + e = -8 \\ 16a + 8b + 4c + 2d + e = 0 \end{matrix}$$

$$\left[\begin{array}{cccc|c} 16 & -8 & 4 & -2 & 28 \\ 1 & -1 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & -6 \\ 1 & 1 & 1 & 1 & -8 \\ 16 & 8 & 4 & 2 & 0 \end{array} \right] \rightarrow \text{rref} \rightarrow \begin{bmatrix} 1 \\ -1 \\ 1 \\ -3 \\ -6 \end{bmatrix}$$

$$x^4 - x^3 + x^2 - 3x - 6 = y$$