MTH 166 Homework #10 Key

1\. a. $z = 4i$ $\quad |z| = 4$

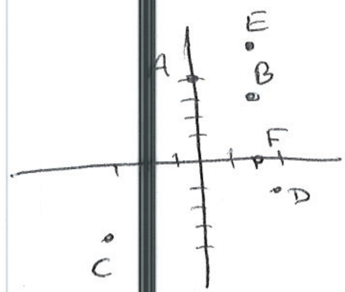
b. $2+3i$ $\quad |z| = \sqrt{2^2+3^2} = \sqrt{13}$

c. $-3-4i$ $\quad |z| = 5$

d. $3-i$ $\quad |z| = \sqrt{10}$

e. $2+5i$ $\quad |z| = \sqrt{29}$

f. 2 $\quad |z| = 2$



2a. $2+2i$ $\quad r = \sqrt{8}$ $\quad \theta = \pi/4$ $\quad \sqrt{8}\left(\cos \pi/4 + i \sin \pi/4\right)$

b. $-2+2i\sqrt{3}$ $\quad r = \sqrt{4+12} = 4$ $\quad \theta = -\pi/3 + \pi = 2\pi/3$ $\quad 4\left(\cos 2\pi/3 + i \sin 2\pi/3\right)$

c. $-2+3i$ $\quad r = \sqrt{13}$ $\quad \theta \approx 2.16$ $\quad \approx \sqrt{13}\left(\cos 2.16 + i \sin 2.16\right)$

d. $1 - i\sqrt{5}$ $\quad r = \sqrt{6}$ $\quad \theta \approx 5.13$ $\quad \approx \sqrt{6}\left(\cos 5.13 + i \sin 5.13\right)$

3a. $6\left(\cos \pi/6 + i \sin \pi/6\right) = 6\left(\frac{\sqrt{3}}{2} + i\left(\frac{1}{2}\right)\right) = 3\sqrt{3} + 3i$

b. $5\left(\cos \pi/2 + i \sin \pi/2\right) = 5(0 + i(1)) = 5i$

c. $8\left(\cos 7\pi/4 + i \sin 7\pi/4\right) = 8\left(1/\sqrt{2} + i\left(-\frac{1}{\sqrt{2}}\right)\right) = 4\sqrt{2} - 4\sqrt{2}i$

d. $20\left(\cos 205° + i \sin 205°\right) \approx 20(-.9063 + -.4226i) = -18.126 - 8.452i$

3a. $z_1 z_2 = 30\left(\cos 70° + i \sin 70°\right)$

$\frac{z_1}{z_2} = \frac{6}{5}\left(\cos(-30°) + i \sin(-30°)\right)$

b. $z_1 z_2 = 12\left(\cos \frac{11\pi}{16} + i \sin 11\pi/16\right)$

$\frac{z_1}{z_2} = \frac{3}{4}\left(\cos 9\pi/16 + i \sin 9\pi/16\right)$

c. $z_1 z_2 = -2$ $\qquad \frac{z_1}{z_2} = \frac{1+i}{-1+i} = -i$

d. $z_1 z_2 = 5 - i$

$\frac{z_1}{z_2} = \frac{1+i}{2-3i} = -\frac{1}{13} + \frac{5}{13}i$

5a. $8(\cos 45° + i\sin 45°) = 4\sqrt{2} + 4\sqrt{2}i$

b. $\frac{1}{8}(\cos 5\pi/3 + i\sin 5\pi/3) = \frac{1}{8}\left(\frac{1}{2} - \frac{\sqrt{3}}{2}i\right) = \frac{1}{16} - \frac{\sqrt{3}}{16}i$

c. $(1+i)^4 = \left[\sqrt{2}(\cos \pi/4 + i\sin \pi/4)\right]^4 = -4$

$= 4(\cos\pi + i\sin\pi)$

d. $243(\cos 5\pi/4 + i\sin 5\pi/4) = -\frac{243}{\sqrt{2}} - \frac{243}{\sqrt{2}}i$

e. $(\sqrt{2} - i)^7 = -13\sqrt{2} + 43i$

6a. $3(\cos 5\pi/6 + i\sin 5\pi/6)$ and $3(\cos 11\pi/6 + i\sin 11\pi/6)$

b. $3(\cos 102° + i\sin 102°)$, $3(\cos 222° + i\sin 222°)$, $3(\cos 342° + i\sin 342°)$

c. $\sqrt{2}(\cos \pi/3 + i\sin \pi/3)$, $\sqrt{2}(\cos 5\pi/6 + i\sin 5\pi/6)$,

$\sqrt{2}(\cos \frac{4\pi}{3} + i\sin \frac{4\pi}{3})$, $\sqrt{2}(\cos 11\pi/6 + i\sin 11\pi/6)$

d. $(1+i) = \sqrt{2}(\cos \pi/4 + i\sin \pi/4)$

$\sqrt[10]{2}(\cos \pi/20 + i\sin \pi/20)$, $\sqrt[10]{2}(\cos 9\pi/20 + i\sin 9\pi/20)$,

$\sqrt[10]{2}(\cos 17\pi/20 + i\sin 17\pi/20)$, $\sqrt[10]{2}(\cos 5\pi/4 + i\sin 5\pi/4)$,

$\sqrt[10]{2}(\cos 33\pi/20 + i\sin 33\pi/20)$

e. $1 = \cos 0 + i\sin 0 = \cos 2\pi + i\sin 2\pi = \cos 4\pi + i\sin 4\pi =$
$\cos 6\pi + i\sin 6\pi = \cos 8\pi + i\sin 8\pi = \cos 10\pi + i\sin 10\pi$

Cube roots: $\cos 0 + i\sin 0 = 1$
$\cos 2\pi/3 + i\sin 2\pi/3 = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$
$\cos 4\pi/3 + i\sin 4\pi/3 = -\frac{1}{2} - \frac{\sqrt{3}}{2}i$

fourth roots $1, i, -1, -i$

fifth roots: $\cos 0 + i\sin 0 = 1$, $\cos 2\pi/5 + i\sin 2\pi/5$,
$\cos 4\pi/5 + i\sin 4\pi/5$, $\cos 6\pi/5 + i\sin 6\pi/5$,
$\cos 8\pi/5 + i\sin 8\pi/5$

6e (cont'd)

Sixth roots: $\cos 0 + i\sin 0 = 1$, $\cos \pi/3 + i\sin \pi/3 = \frac{1}{2} + \frac{\sqrt{3}}{2}i$

$\cos 2\pi/3 + i\sin 2\pi/3 = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$, $\cos\pi + i\sin\pi = -1$,

$\cos 4\pi/3 + i\sin 4\pi/3 = -\frac{1}{2} - \frac{\sqrt{3}}{2}i$, $\cos 5\pi/3 + i\sin 5\pi/3 = \frac{1}{2} - \frac{\sqrt{3}}{2}i$

f. $-i = \cos 3\pi/2 + i\sin 3\pi/2 = \cos 7\pi/2 + i\sin 7\pi/2 = \cos 11\pi/2 + i\sin 11\pi/2$

$= \cos 15\pi/2 + i\sin 15\pi/2 = \cos 19\pi/2 + i\sin 19\pi/2 = \cos 23\pi/2 + i\sin 23\pi/2$

Cube roots: $\cos \pi/2 + i\sin\pi/2 = i$, $\cos 7\pi/6 + i\sin 7\pi/6 = -\frac{\sqrt{3}}{2} - \frac{1}{2}i$

$\cos 11\pi/6 + i\sin 11\pi/6 = \frac{\sqrt{3}}{2} - \frac{1}{2}i$

fourth roots: $\cos 3\pi/8 + i\sin 3\pi/8$, $\cos 7\pi/8 + i\sin 7\pi/8$,

$\cos 11\pi/8 + i\sin 11\pi/8$, $\cos 15\pi/8 + i\sin 15\pi/8$

fifth roots: $\cos 3\pi/10 + i\sin 3\pi/10$, $\cos 7\pi/10 + i\sin 7\pi/10$,

$\cos 11\pi/10 + i\sin 11\pi/10$, $\cos 3\pi/2 + i\sin 3\pi/2$,

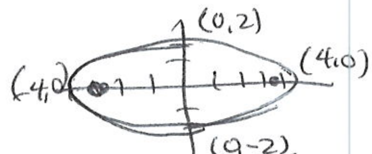
$\cos 19\pi/10 + i\sin 19\pi/10$

sixth roots: $\cos \pi/4 + i\sin\pi/4$, $\cos 7\pi/12 + i\sin 7\pi/12$,

$\cos 11\pi/12 + i\sin 11\pi/12$, $\cos 15\pi/12 + i\sin 15\pi/2$

$\cos 19\pi/12 + i\sin 19\pi/12$, $\cos 23\pi/12 + i\sin 23\pi/12$

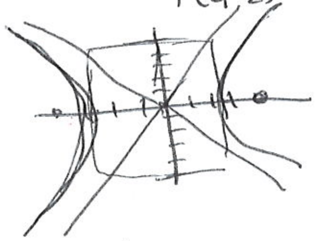
7.a. $\frac{x^2}{16} + \frac{y^2}{4} = 1$



$c = 16 - 4 = \sqrt{12}$

$(\sqrt{12},0)$ $(-\sqrt{12},0)$ foci

b. $\frac{x^2}{8} - \frac{y^2}{25} = 1$



$8 + 25 = 31$

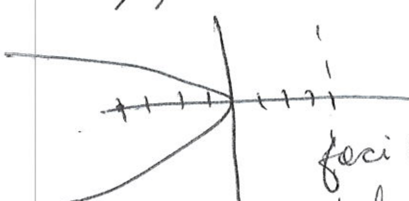
$c = \sqrt{31}$

foci $(\pm\sqrt{31}, 0)$

vertices $(\pm\sqrt{8}, 0)$

asymptotes $y = \frac{\pm 5}{2\sqrt{2}}x$

c. $y^2 = -8x$

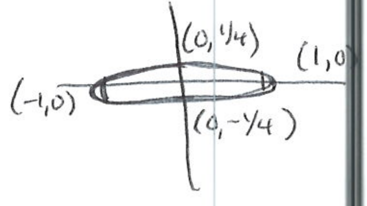


foci $(-4,0)$

directrix $x = 4$

d. $x^2 = 1 - 4y^2$

$x^2 + 4y^2 = 1 \Rightarrow x^2 + \frac{y^2}{(1/4)} = 1$



$1 - 1/4 = \frac{\sqrt{3}}{2}$

foci $(\pm\frac{\sqrt{3}}{2}, 0)$