MAT 166, Final Exam, Fall 2017 Name KEY

Instructions: Show all work. Give exact answers unless specifically asked to round. Complete all parts of each question. Questions that provide only answers and no work will not receive full credit. If you use your calculator (only when problems don't instruct you to do the problem by hand), showing calculator steps will count as "work".

1. Solve the system $\begin{cases} 5x + 12y + z = 10 \\ 2x + 5y + 2z = -1 \\ x + 2y - 3z = 5 \end{cases}$ by any method. (12 points)

Solution methods will vary

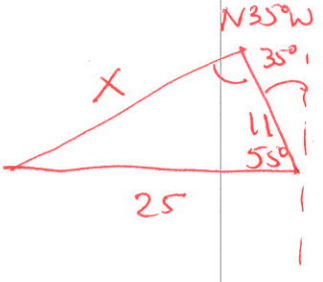
$$\left[\begin{array}{ccc|c} 5 & 12 & 1 & 10 \\ 2 & 5 & 2 & -1 \\ 1 & 2 & -3 & 5 \end{array}\right]$$

$\Rightarrow$ rref $\Rightarrow$

$$\left[\begin{array}{ccc|c} 1 & 0 & -19 & 0 \\ 0 & 1 & 8 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

The system is inconsistent (no solution)

2. You are on a fishing boat that leaves its pier and heads east. After traveling for 25 miles, there is a report of rough seas directly north, so the captain turns the boat to a bearing of $N35°W$ for 11 miles. How far is the boat to the pier, and in which direction would the boat have to sail in order to read port from their current position? (10 points)



$X^2 = 25^2 + 11^2 - 2(25)(11)\cos 55°$

$X = 20.75$ miles

bearing S 44.3° W

$\frac{\sin 55°}{20.75} = \frac{\sin B}{25}$

$\sin B = .98693$

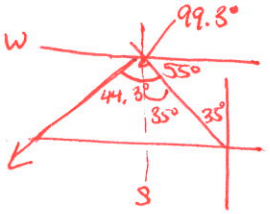
$B = 80.73°$

or $99.27°$

$\cos B = \frac{25^2 - 20.75^2 - 11^2}{-2(20.75)(11)}$

$\cos B = -.16087$

$B = 99.26°$



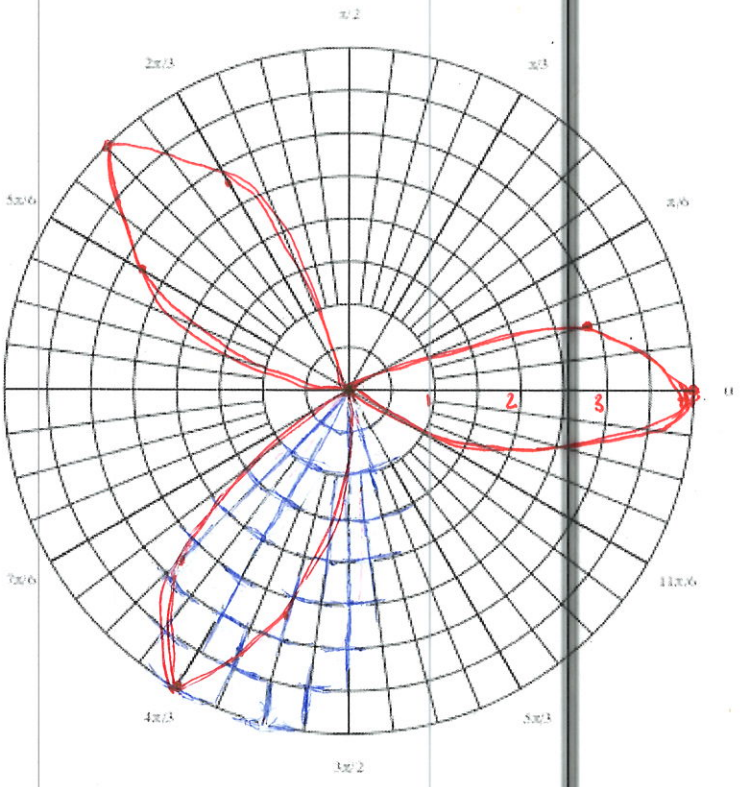
3. Convert the equation $\theta = \frac{2\pi}{3}$ into rectangular coordinates. (8 points)

$\tan^{-1}(y/x) = \theta$

$\tan\left(\frac{2\pi}{3}\right) = y/x$

$-\sqrt{3} = y/x \Rightarrow \boxed{y = -\sqrt{3}x}$

4. Graph the polar equation $r = 4\cos 3\theta$ on the polar graph below. Clearly label at least 6 points and show work. (12 points)



θ	r
0	4
$\pi/6$	0
$\pi/4$	-2.828
$\pi/3$	-4
$\pi/2$	-2.828
$2\pi/3$	0
$3\pi/4$	4
$5\pi/6$	2.828
π	0

5. Find $(-1+i)^7$ using DeMoivre's Theorem. (You will no receive credit for FOILing.) Write the result in standard form with exact values. (12 points)

$r = \sqrt{(-1)^2+1^2} = \sqrt{2} \qquad \theta = 3\pi/4$

$(\sqrt{2})^7\left(\cos 21\pi/4 + i\sin 21\pi/4\right) = (\sqrt{2})^7\left(-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i\right)$

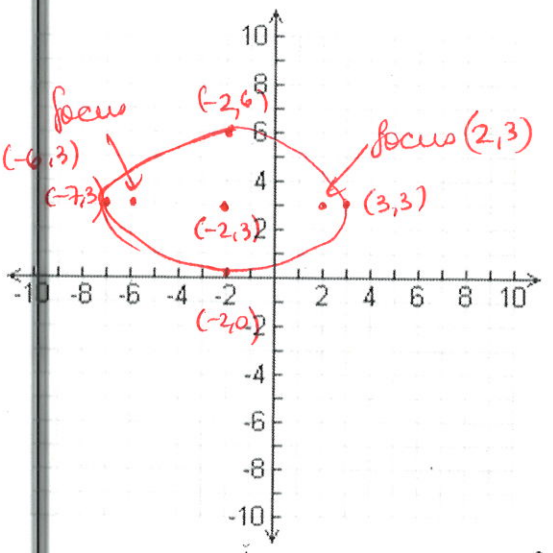
$\sqrt{2}\cdot(\sqrt{2})^6$

$= 8(-1-i) = \boxed{-8-8i}$

6. Graph the equation $\frac{(x+2)^2}{25} + \frac{(y-3)^2}{9} = 1$ on the axes below. Clearly label the foci, vertices and minor axis endpoints. (10 points)

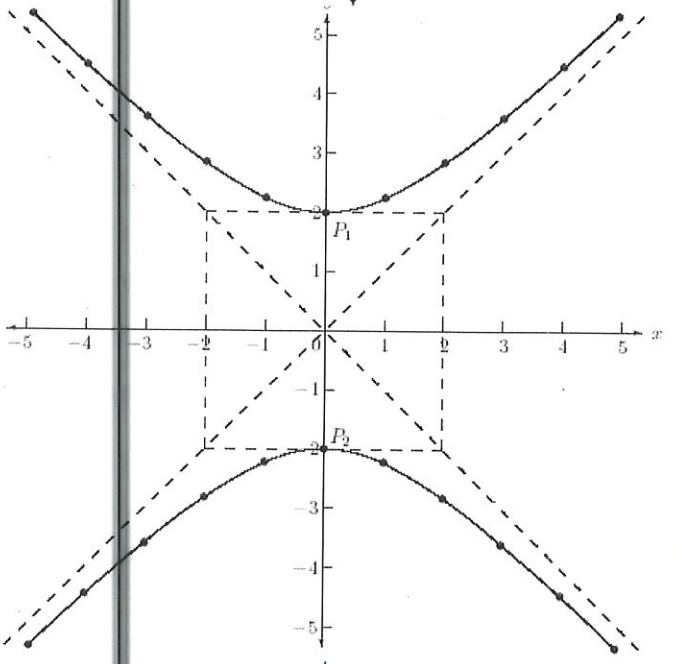
$25 - 9 = 16$

$c = 4$



7. The graph of a hyperbola is shown below. Write the equation of the graph in standard form. (10 points)

$\frac{y^2}{4} - \frac{x^2}{4} = 1$



8. Sketch the graph of the parametric equations $x = 2t + 3, y = -3t + 1$, by plotting at least 4 points (and labeling them). Use an arrow to indicate the orientation of time. Then convert the equation back to an equation in x and y only. (10 points)

t	x	y
-3	-3	10
-2	-1	7
-1	1	4
0	3	1
1	5	-2
2	7	-5
3	9	-8

$\frac{x-3}{2} = t$

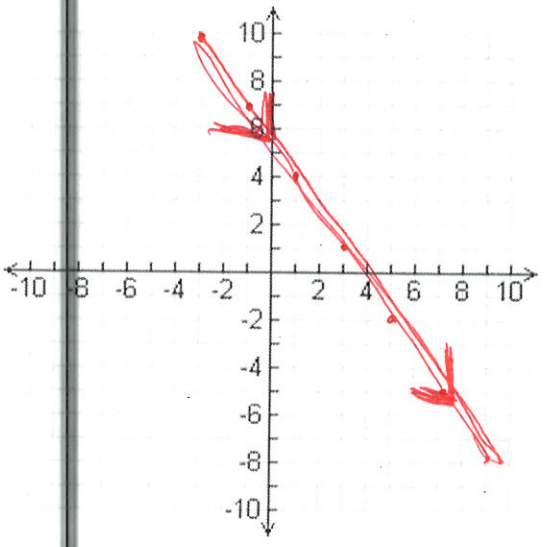
$\frac{y-1}{-3} = t$

$\frac{x-3}{2} = \frac{y-1}{-3}$

$-3(x-3) = 2(y-1)$

$-3x + 9 = 2y - 2$

$-3x + 11 = 2y$ $\Rightarrow$ $y = -\frac{3}{2}x + 11/2$



9. Evaluate the following expressions. (8 points each)

a. $\frac{20!}{4!16!}$

4845

b. $\sum_{k=1}^{4}(k-3)(k+2)$

$(-2)(3) + (-1)(4) + (0)(5) + (1)(6)$

$-6 - 4 + 0 + 6 = -4$

c. $\binom{15}{2}$

105

10. Write the sum of $1 + 8 + 27 + 64 + 125 + \cdots + 729$ in summation notation. (8 points)

$\sum_{k=1}^{9} k^3$

11. Use the binomial theorem to expand $(2y - 3)^4$. (10 points)

$(2y)^4 + 4(2y)^3(-3) + 6(2y)^2(-3)^2 + 4(2y)(-3)^3 + (-3)^4$

$16y^4 - 96y^3 + 216y^2 - 216y + 81$