By Parts

a i. $u = \operatorname{arcsec} x$  $dv = dx$

ii. $u = \ln x$  $dv = x\,dx$

iii. $u = x^2$  $dv = x\sin(x^2)\,dx$

iv. $u = x$  $dv = (3x+4)^{-1/2}\,dx$

v. $u = \cos(x)$  $dv = e^{4x}\,dx$

b. vi. $\int \frac{x^2}{\sqrt[3]{2x+1}}\,dx$

| | | |
|---|---|---|
| $+$ | $u = x^2$ | $dv = (2x+1)^{-1/3}\,dx$ |
| $-$ | $2x$ | $\frac{1}{2}\cdot\frac{3}{2}(2x+1)^{2/3} = \frac{3}{4}(2x+1)^{2/3}$ |
| $+$ | $2$ | $\frac{1}{2}\frac{3}{4}\cdot\frac{3}{5}(2x+1)^{5/3} = \frac{9}{40}(2x+1)^{5/3}$ |
| $-$ | $0$ | $\frac{1}{2}\frac{3}{8}\cdot\frac{9}{40}(2x+1)^{8/3} = \frac{27}{640}(2x+1)^{8/3}$ |

$$\frac{3}{4}x^2(2x+1)^{2/3} - \frac{9}{20}x(2x+1)^{5/3} + \frac{27}{320}(2x+1)^{8/3} + C$$

vii. $\int x^2 e^{-3x}\,dx$

| | | |
|---|---|---|
| $+$ | $u = x^2$ | $dv = e^{-3x}\,dx$ |
| $-$ | $2x$ | $-\frac{1}{3}e^{-3x}$ |
| $+$ | $2$ | $\frac{1}{9}e^{-3x}$ |
| $-$ | $0$ | $-\frac{1}{27}e^{-3x}$ |

$$-\frac{1}{3}x^2e^{-3x} - \frac{2}{9}xe^{-3x} - \frac{2}{27}e^{-3x} + C$$

viii. $\int x^2 \arcsin(x)\,dx$

$u = \arcsin x$  $dv = x^2$

$du = \frac{1}{\sqrt{1-x^2}}\,dx$  $v = \frac{1}{3}x^3$

$$\frac{1}{3}x^3\arcsin(x) - \frac{1}{3}\int\frac{x^3}{\sqrt{1-x^2}}\,dx$$

$u = x^2$  $dv = \frac{x}{\sqrt{1-x^2}}\,dx = x(1-x^2)^{-1/2}\,dx$

$du = 2x\,dx$  $v = -(1-x^2)^{1/2}$

$\frac{1}{3}x^3\arcsin x - \frac{1}{3}\left[-x^2(1-x^2)^{1/2} - \int -2x(1-x^2)^{1/2}\,dx\right]$

$= \frac{1}{3}x^3\arcsin x + \frac{1}{3}x^2\sqrt{1-x^2} - \frac{2}{3}\int x(1-x^2)^{1/2}\,dx$

$u = 1-x^2 \quad du = -2x\,dx$

$-\frac{1}{2}du = x\,dx$

$= \frac{1}{3}x^3\arcsin x + \frac{1}{3}x^2\sqrt{1-x^2} + \frac{1}{3}\int u^{1/2}\,du$

$= \frac{1}{3}x^3\arcsin x + \frac{1}{3}x^2\sqrt{1-x^2} + \frac{1}{3}\cdot\frac{2}{3}(1-x^2)^{3/2} + C$

$= \frac{1}{3}x^3\arcsin x + \frac{1}{3}x^2\sqrt{1-x^2} + \frac{2}{9}(1-x^2)^{3/2} + C$

ix. $\int x\sec^2 2x\,dx$

$u = x \qquad dv = \sec^2(2x)\,dx$

$du = dx \qquad v = \frac{1}{2}\tan(2x)$

$\frac{1}{2}x\tan(2x) - \int \frac{1}{2}\tan(2x)\,dx$

$= \frac{1}{2}x\tan(2x) + \frac{1}{4}\ln|\cos(2x)| + C$

x. $\int \frac{xe^x}{(x+1)^2}\,dx$

$u = xe^x \qquad dv = \frac{1}{(x+1)^2}\,dx$

$du = e^x + xe^x = e^x(1+x) \qquad v = \frac{-1}{x+1}$

$\frac{-xe^x}{x+1} - \int -\frac{e^x(x+1)}{x+1}\,dx$

$\frac{-xe^x}{x+1} + \int e^x\,dx = -\frac{xe^x}{x+1} + e^x + C$

xi. $\int \cos(3x)e^{-4x}\,dx$

$u = \cos 3x \qquad dv = e^{-4x}\,dx$

$du = -3\sin(3x)\,dx \qquad v = -\frac{1}{4}e^{-4x}$

$-\frac{1}{4}\cos(3x)e^{-4x} - \int +\frac{3}{4}e^{-4x}\sin(3x)\,dx$

$-\frac{1}{4}e^{-4x}\cos(3x) - \frac{3}{4}\int e^{-4x}\sin(3x)\,dx$

$u = \sin 3x \qquad dv = e^{-4x}\,dx$

$du = 3\cos 3x\,dx \qquad v = -\frac{1}{4}e^{-4x}$

$$-\frac{1}{4}e^{-4x}\cos(3x) - \frac{3}{4}\left[-\frac{1}{4}e^{-4x}\sin 3x - \int -\frac{1}{4}e^{-4x}\cdot 3\cos 3x\,dx\right] =$$

$$\frac{16}{16}\int \cos(3x)e^{-4x}dx = -\frac{1}{4}e^{-4x}\cos 3x + \frac{3}{16}e^{-4x}\sin 3x - \frac{9}{16}\int e^{-4x}\cos(3x)\,dx$$

$$+\frac{9}{16}\int \cos(3x)e^{-4x}\,dx \qquad\qquad +\frac{9}{16}\int e^{-4x}\cos(3x)\,dx$$

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$$\frac{25}{16}\int \cos(3x)e^{-4x}dx = \left[-\frac{1}{4}e^{-4x}\cos 3x + \frac{3}{16}e^{-4x}\sin 3x + C\right]\cdot\frac{16}{25}$$

$$\int \cos(3x)e^{-4x}dx = \frac{-4}{25}e^{-4x}\cos 3x + \frac{3}{25}e^{-4x}\sin 3x + C$$

xii. $\int (2x-1)^4\cos(5x)\,dx$

| | | |
|---|---|---|
| + | $u = (2x-1)^4$ | $dv = \cos(5x)\,dx$ |
| − | $8(2x-1)^3$ | $\frac{1}{5}\sin(5x)$ |
| + | $48(2x-1)^2$ | $-\frac{1}{25}\cos 5x$ |
| − | $192(2x-1)$ | $\frac{-1}{125}\sin 5x$ |
| + | $384$ | $\frac{1}{625}\cos 5x$ |
| − | $0$ | $\frac{1}{3125}\sin 5x$ |

$$\frac{1}{5}(2x-1)^4\sin 5x + \frac{8}{25}(2x-1)^3\cos 5x - \frac{48}{125}(2x-1)^2\sin 5x - \frac{192}{625}(2x-1)\cos 5x$$

$$+\frac{384}{3125}\sin 5x + C$$